Investigation

Complete the Square

You can use rectangle diagrams to help convert from general form to vertex form.

Step 1 Consider the expression $x^2 + 6x$.

a. What could you add to the expression to make it a perfect square? That is, what must you add to complete this rectangle diagram?

	x	3
x	x^2	$3x$
3	$3x$	$\underline{?}$

b. If you add a number to an expression, then you must also subtract the same amount in order to preserve the value of the original expression. Fill in the blanks to rewrite $x^2 + 6x$ as the difference between a perfect square and a number.

$x^2 + 6x = x^2 + 6x + \underline{?} - \underline{?} = (x + 3)^2 - \underline{?}$

c. Use a graph or table to verify that your expression in the form $(x - h)^2 + k$ is equivalent to the original expression, $x^2 + 6x$.

Step 2 Consider the expression $x^2 + 6x - 4$.

a. Focus on the 2nd- and 1st-degree terms of the expression, $x^2 + 6x$. What must be added to and subtracted from these terms to complete a perfect square yet preserve the value of the expression?

	x	3
x	x^2	$3x$
3	$3x$	$\underline{?}$

b. Rewrite the expression $x^2 + 6x - 4$ in the form $(x - h)^2 + k$.

c. Use a graph or table to verify that your expression is equivalent to the original expression, $x^2 + 6x - 4$.

Step 3 Rewrite each expression in the form $(x - h)^2 + k$. If you use a rectangle diagram, focus on the 2nd- and 1st-degree terms first. Verify that your expression is equivalent to the original expression.

a. $x^2 - 14x + 3$ **b.** $x^2 - bx + 10$

When the 2nd-degree term has a coefficient, you can first factor it out of the 2nd- and 1st-degree terms. For example, $3x^2 + 24x + 5$ can be written $3(x^2 + 8x) + 5$. Completing a diagram for $x^2 + 8x$ can help you rewrite the expression in the form $a(x - h)^2 + k$.

	x	4
x	x^2	$4x$
4	$4x$	16

$3x^2 + 24x + 5$	The original expression.
$3(x^2 + 8x) + 5$	Factor the 2nd- and 1st-degree terms.
$3(x^2 + 8x + 16) - 3(16) + 5$	Complete the square. You add $3 \cdot 16$, so you must subtract $3 \cdot 16$.
$3(x + 4)^2 - 43$	An equivalent expression in the form $a(x - h)^2 + k$.

Step 4

Rewrite each expression in the form $a(x - h)^2 + k$. Use a graph or table to verify that your expression is equivalent to the original expression.

a. $2x^2 - 6x + 1$ **b.** $ax^2 + 10x + 7$

Step 5

Use the strategy from Step 4 to rewrite this expression in the form $a(x - h)^2 + k$:

$ax^2 + bx + c$

Step 6

If you graph the quadratic function $y = ax^2 + bx + c$, what will be the coordinates of the vertex in terms of a, b, and c? Why?